Probability: It's Not as Hard as it Sounds

Ben Babcock
University of Minnesota

The Basic Idea

Probability quantifies the chances that an event will occur based on information about the event.

It is a number between _ and _.

$p(\text{A}) = \text{x}$

“The probability that A occurs is x.”

Why do you think that the mathematics of probability was invented?

Example: M & M's®

Suppose that Jack Bauer has a small bag of M & M's candies with 6 blue, 8 orange, and 10 brown. If he draws randomly from the bag, what is the probability that he will draw a brown candy?

Analytic approach: H, 125-126

Total number of possible events: 10 + 8 + 6 = 24

An event is any outcome of the trial.

Total number of possible successes for 1 draw: 10

A success is what we "want" to happen.

Probability of success: $p(\text{brown}) = 10 / 24 = 0.417$

(note the decimal format)

Example: M & M's®

Relative frequency approach: H, 126

Jack forces a terrorist to draw an M & M, record its color, and return it to the bag (<u>sampling with replacement</u>). After a lot of draws, the terrorist gives Jack the information that he wants by reporting the proportion of browns he drew.

Types of Events

There are two types of events that people often talk about in probability and statistics: <u>independent events</u> and <u>mutually exclusive events</u>.

<u>Mutually exclusive</u>: If event A occurs, event B _________ occur.
Example: John Doe passing PSY 2801 and John Doe failing PSY 2801 are mutually exclusive.

<u>Independent</u>: Event A does not ____________ event B; one, both, or neither can occur.
Example 1: John Doe passing PSY 2801 and Jane Doe passing PSY 2801.
H, 128

Laws of Probability

Additive law of probability: To find the probability of either *event A or event B* occurring, simply add their individual probabilities. H, 129. Give me an example.

This only works with MUTUALLY EXCLUSIVE events!

Multiplicative law of probability: To find the probability of both *event A and event B* occurring, simply multiply their individual probabilities. H, 129. Give me an example. H, 129-130. Give an example.

This only works with INDEPENDENT events!

Multiplying the probabilities of independent events gives you the __________ probability of both events occurring.

Conditional Probability

A <u>conditional probability</u> is the probability that an event will occur __________ that something else has already occurred.
$p(B|A) = x$
"The probability of B _________ A is x."

How does conditional probability relate to independent events?

To mutually exclusive events?

CONDITIONAL PROBABILITY IS THE SINGLE MOST IMPORTANT PROBABILITY CONCEPT FOR YOU TO UNDERSTAND FOR THIS STATISTICS CLASS!!!!!!!

A *p*-value is a conditional probability.

Conditional Probability and Bayes' Theorem

Rev. Thomas Bayes (1702-1761) had only one original mathematics work published, and that was after he died. This work contains one of the most important theorems in probability and statistics.

Bayes' Theorem:

$$p(B|A) = \frac{p(A|B) \times p(B)}{p(A)}$$

where A and B are events.

How is this useful?

Conditional Probability and Bayes' Theorem

Suppose that you wanted to go to a Minnesota Twins game. We do not know if the Twins are at home. Word on the street is that Joe Mauer is going to play catcher today, and the manager likes to play Mauer at catcher when the Twins are at home. Reasons:

Great swing, solid fielding
Hometown hero
Just so darn cute

What we want to know:
P(Twins are at home | Mauer at catcher)

What we know:
P(Mauer at catcher) = .6,   *P*(Twins are at home) = .5
P(Mauer at catcher | Twins are at Home) = .8

Conditional Probability and Bayes' Theorem

Bayes' Theorem:

$$p(B|A)=\frac{p(A|B)\times p(B)}{p(A)}$$

What we know:
P(Mauer at catcher) = .6
P(Twins are at home) = .5
P(Mauer at catcher | Twins are at home) = .8
Calculations:

Next time, we'll check the newspaper.

Odds

The odds of an event are simply the probability of the event occurring divided by the probability of an event not occurring.

$$odds(A) = \frac{P(A)}{1 - P(A)}$$

where A is an event and
P is the probability of an event.

Why use odds?

- Odds have no upper bound.
- Odds have nicer mathematical properties than probabilities.
- The odds ratio is used in many statistics.

Probability vs. Density

Some sets of events can only take on discrete values. Each of the events has a probability value associated with it.

Event set: Whether a baby is born biologically male, female, or other.

p(female) = 0.510
p(male) = 0.485
p(other) = 0.005

H, 136

Probability vs. Density

Some sets of events can take on an "infinite" number of values. Each of the events has a density value associated with it.

A density is not a probability, because the probability of a person having a given score is mathematically ______.

Example: What is the probability that a woman is *exactly* 5 feet 3 inches tall?

Why is the probability of this occurring always _______?

Probability vs. Density

You find probability from a density by finding the ________ under the _________. A ***range*** of values has a probability value. H, 137-138.

Example: What is the probability that a woman is between 5 feet 3 inches and 5 feet 7 inches tall?

If female height was normally distributed, we could use the normal curve to obtain the approximate probability.