

# Two-Sample $t$ -tests

Ben Babcock  
University of Minnesota

# Two Samples: Pairing the Samples

Suppose you wanted to study a population with natural pairings.

There is a  $t$ -test made for this situation. It is the paired samples  $t$ -test. The equation for the paired samples  $t$ -test is

$$t(df) = \frac{\bar{D} - \text{Null } \bar{D}}{s_D / \sqrt{N}}$$

where  $D$  is a difference score

$\bar{D}$  is the mean of the difference scores and

$N$  is number of pairs, and  $df$  is the degrees of freedom ( $N-1$ ).

Difference score: for each pair, person 1's score minus person 2's score.

# Paired Samples $t$ -test: Steps of the Statistic

$$t(df) = \frac{\bar{D} - \text{Null } \bar{D}}{s_D / \sqrt{N}}$$

So we need to:

1. formulate our hypotheses and set our critical alpha level.
2. gather data from every person.
3. obtain a difference score for each pair.
4. find the mean and sd of the difference scores.
5. calculate the statistic.
6. find a  $p$ -value based on the statistic and  $df$ .
7. make a decision based on the  $p$ -value.

Howell works an example on pg. 315

# Paired Samples $t$ -test: Assumptions

1. Normality of difference scores. You assume that the difference scores in the *population* are normally distributed.

Violation Consequence:

Solution:

2. Independence of pairs of observations. This means that one pair of participants (people, rats, trees, etc.) does not influence other pairs of participants.

Violation Consequence:

Solution:

# Should I Use It?

If you have natural pairings, then yes.

It gives you more \_\_\_\_\_.

Research situations where it could be useful:

Problems:

1. Sampling:

2. Difference scores are generally unreliable.

# Research Situation

Suppose that there was a new drug developed that is supposed to help depression. Researchers want to see how this drug performs compared to Zoloft<sup>®</sup>, a leading depression drug. What should we do?

How could we use the 1-sample  $t$ -test?

Will the paired sample (dependent sample)  $t$ -test work?

# Independent Samples $t$ -test

Suppose that we have two non-linked samples (e.g. control and experimental, old drug and new drug, male and female).

We want to know if the two groups have a significant

\_\_\_\_\_.

We should use the Independent Samples  $t$ -test.

$$\textit{Statistic} = \frac{\textit{Mean Difference}}{\textit{Standard Error}}$$

# Independent Samples $t$ -test: Equations

When group sample sizes are equal:

$$t(df) = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n} + \frac{s_2^2}{n}}}$$

where  $n$  is the number of people in each group, subscripts 1 and 2 represent two independent groups, and  $\mu_1 - \mu_2$  is the null hypotheses mean difference.

$$N = 2n$$

Degrees of freedom: \_\_\_\_\_

# Independent Samples $t$ -test: Equations

When group sample sizes are unequal:

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{s_p^2 \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}}$$

where  $n$  is the number of people in the different groups and  $s_p^2$  is the pooled variance.

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

The pooled variance is a type of weighted \_\_\_\_\_.

Degrees of freedom: \_\_\_\_\_

# Research Situation

Suppose that there was a new drug developed that is supposed to help depression. Researchers want to see how this drug performs compared to Zoloft<sup>®</sup>, a leading depression drug.

1. Hypotheses:

Critical alpha:

2. Sampling:

Assignment:

3. Administer drug therapy and gather data.

4. Find the mean and sd for both groups

5. Calculate the independent samples  $t$ -test. Example on board.

6. Find a  $p$ -value based on this statistic.

7. Draw a conclusion about the manipulation/groups.

# Independent Samples $t$ -test: Assumptions

1. Normality. You assume that the scores in the *populations* of the individual groups are normally distributed.

Violation Consequence:

Solution:

2. Homogeneity of variance. You assume that the variances of the two *populations* are equal.

Violation Consequence:

Solution:

3. Independence of observations. This means that one participant (person, rat, tree, etc.) does not influence another participant.

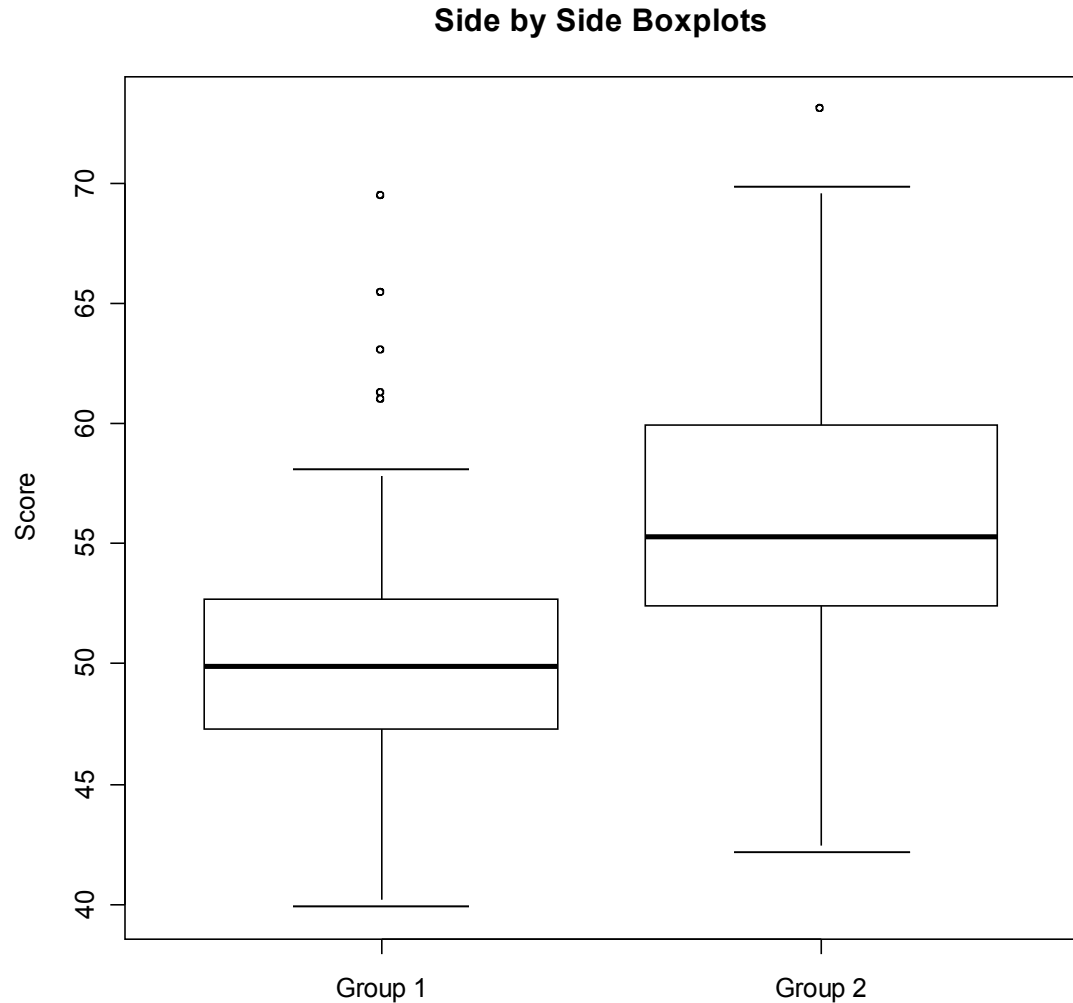
Violation Consequence:

Solution:

# Independent Samples $t$ -test: A Nifty Plot

Side-by-side boxplots of the data are very informative in the context of a  $t$ -test.

You can examine skewness and the midspreads of the groups.



# *t*-test applet

Robustness (or lack thereof) of *t*-test to violations of normality and homogeneity of variance.

[http://www.ruf.rice.edu/%7Elane/stat\\_sim/robustness/index.html](http://www.ruf.rice.edu/%7Elane/stat_sim/robustness/index.html)

Note how the long term *p*-values (type I error rate) change as you manipulate sample size, population variance, and skew.