The Analysis of Variance
ANOVA

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Two Samples to More Samples

It would be nice to be able to compare more than two groups that have different levels of an independent variable.
Examples:

We must take a slightly different approach to a familiar problem.
The conceptual equation

$$Statistic = \frac{Mean\ Difference}{Standard\ Error}$$

is a specific form of the more general conceptual equation

$$Statistic = \frac{Effect}{Error}$$

Null Hypothesis: $\mu_1 = \mu_2 = \mu_3 = \ldots = \mu_j$ H, 377

ANOVA: A Confusing Table

We are working toward filling in this table:

Source	Sum of Sq.	df	Mean Sq.	F
Between	x	x	x	x
Within	x	x	x	
Total	x	x		

This table is confusing at first. If you can fill in a few things, the rest falls into place.
Between = Effect
Within = Error

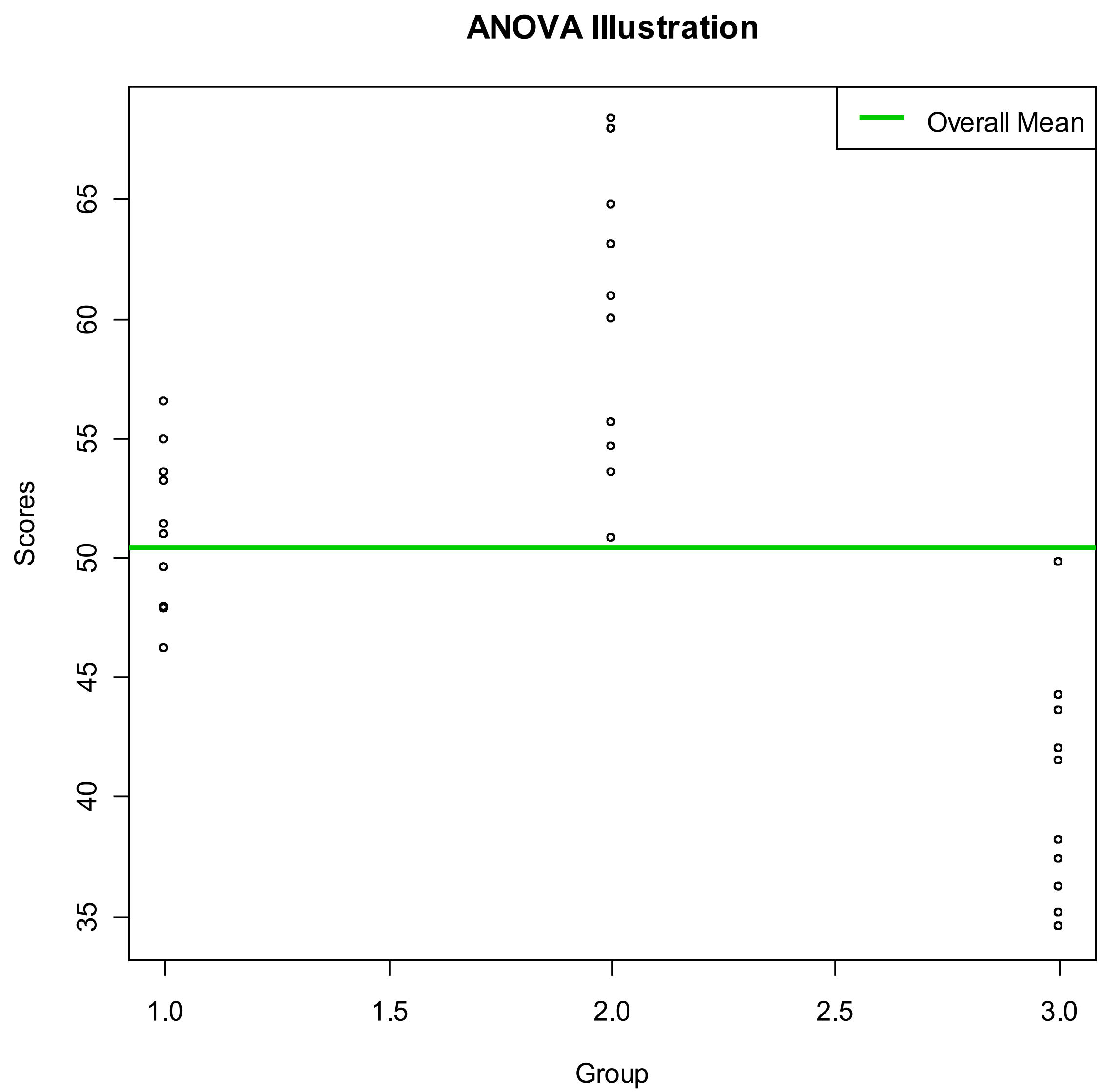
ANOVA: The Basic Premise

We have 3 or more means, so why nut just use several *t*-tests?

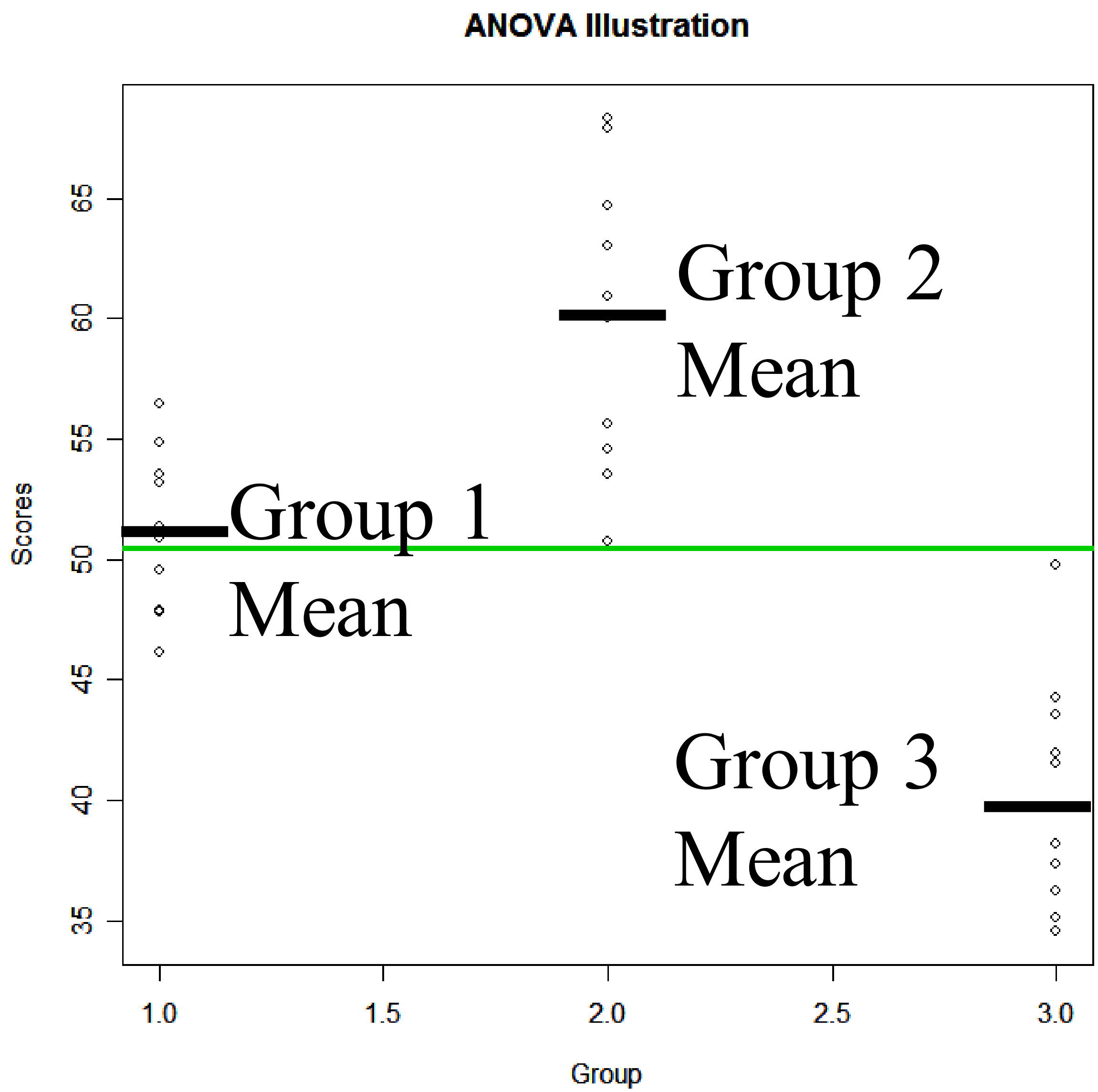
We want to find the variation in scores due to group membership (between groups) and the variation of scores due to error (within groups). If the ratio of membership variance to error variance is sufficiently large, then we will ____________________________.

The null hypothesis for the omnibus (overall) ANOVA is:
H_0: $\mu_1 = \mu_2 = \ldots = \mu_g$
where g is the number of groups.
The alternative hypothesis:

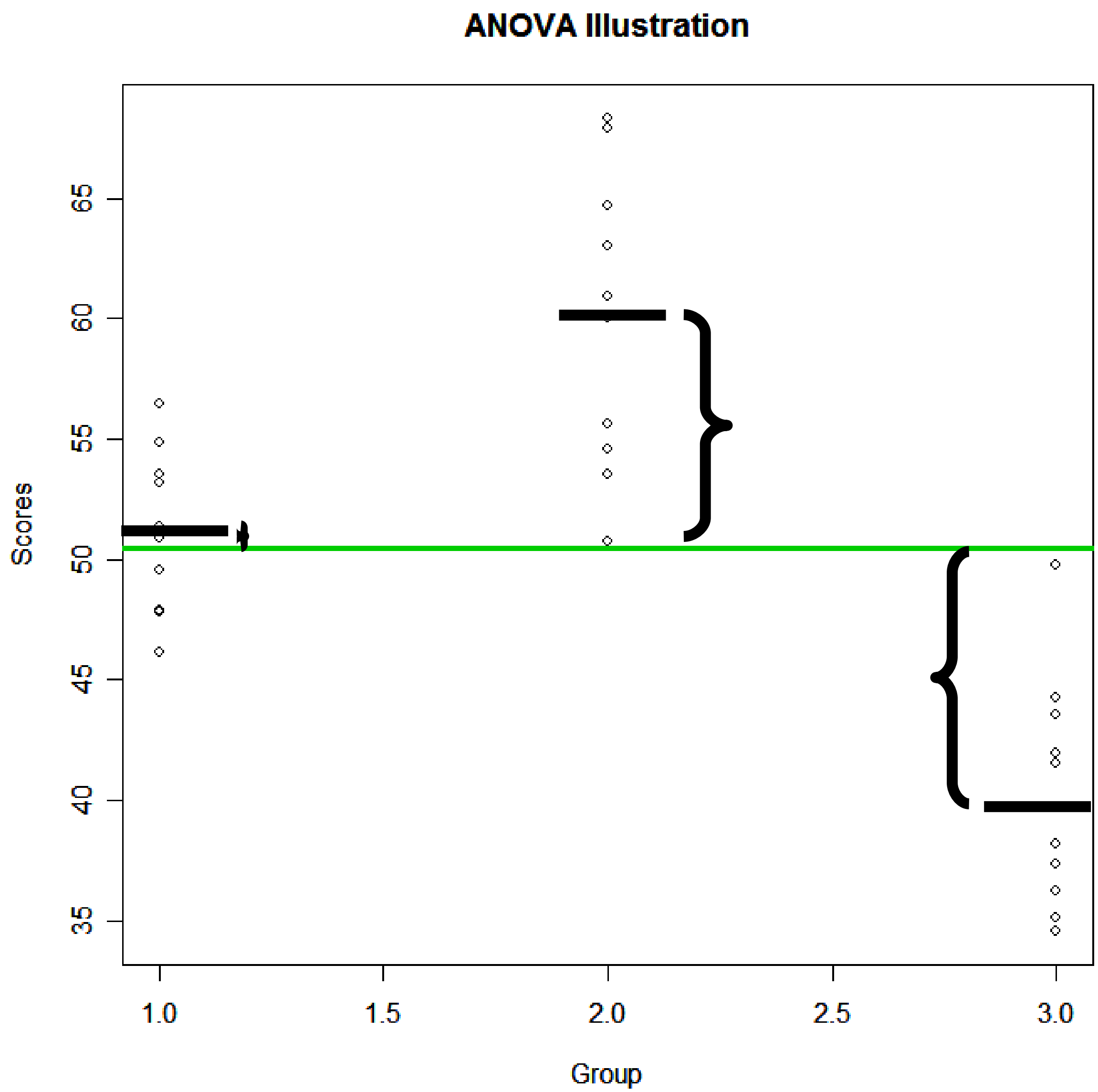
ANOVA: An Illustration



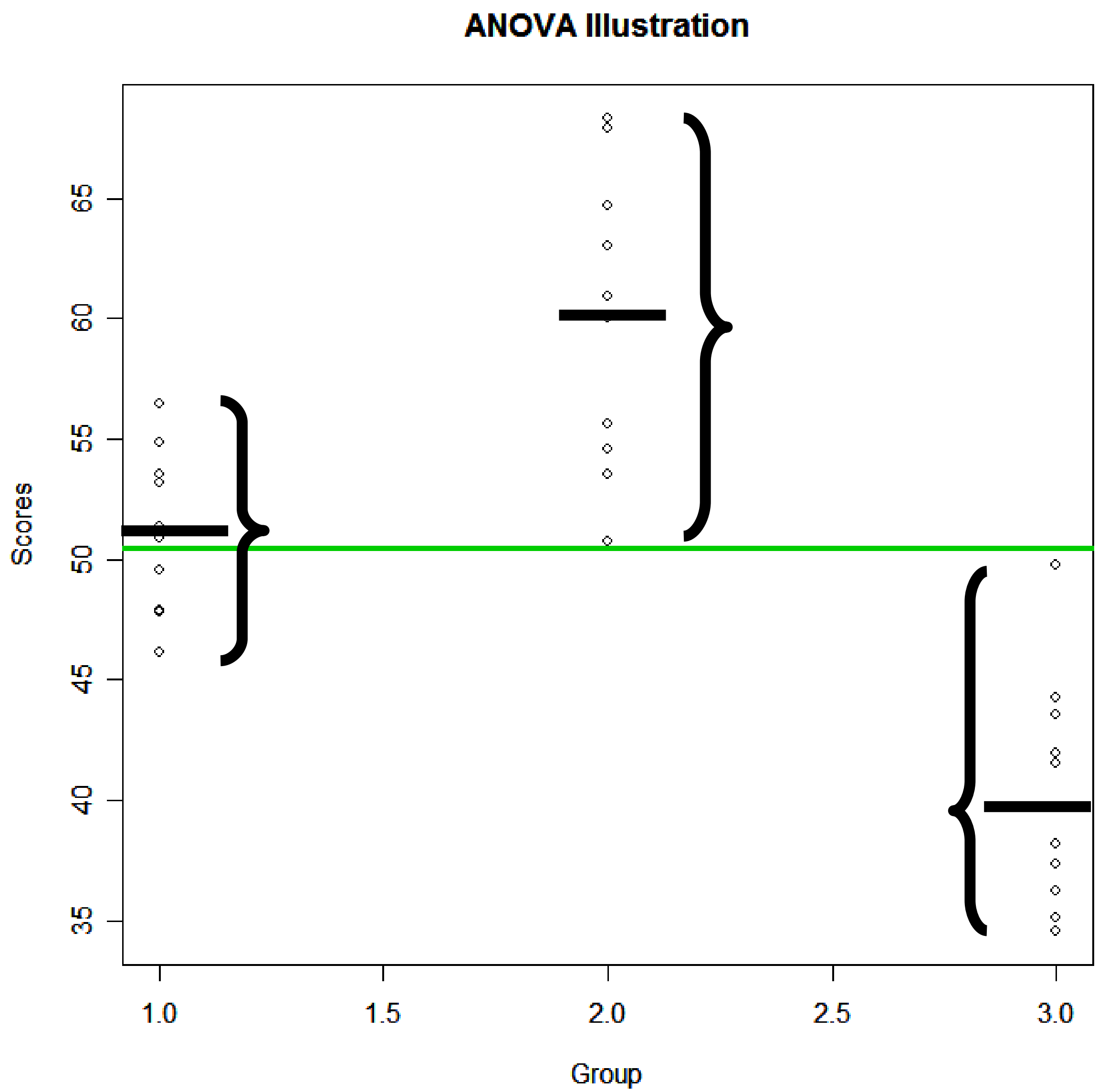
ANOVA: An Illustration



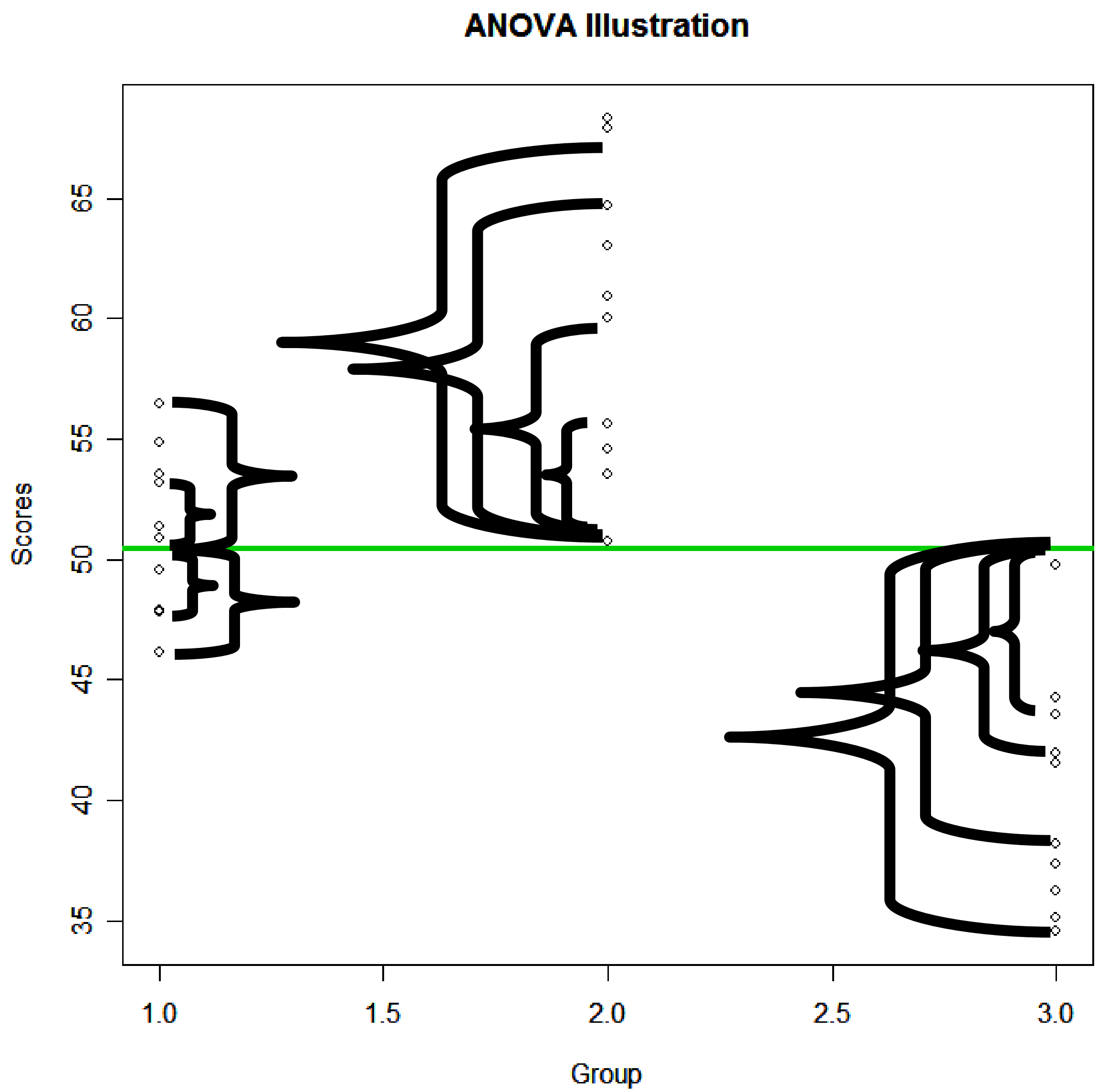
ANOVA: SS Between (Unscaled Effect)



ANOVA: SS Within (Unscaled Error)



ANOVA: SS Total



ANOVA: The Calculation

Ignore Howell 379 – 383.

Sum of squares total: Take an individual score and subtract it from the grand mean. Square that score. Sum for all scores. This is the sum of squared dev. scores from the ______________.

Sum of squares between (equal group sizes): Take a group mean and subtract the grand mean. Square that score. Sum for all groups. Multiply that sum by the sample size of a group. This is a sum of squared deviation scores for the __________________________ from the grand mean.

Sum of squares within (error): SSTotal - SSBetween

ANOVA: Degrees of Freedom

The df Total is simply the total sample size _______________.
The df Between is the total number of groups ________________.
The df Within is the df Total minus df Between.

Notice that our calculation strategy of df is the same the strategy for calculating the sums of squares:
Calculate the Total and Between figures. Within has to be the leftovers.

Mean square for Between and Within:
Divide the sum of squares by the degrees of freedom.

Hmmm...dividing a sum of squared deviation scores by something that's like sample size...

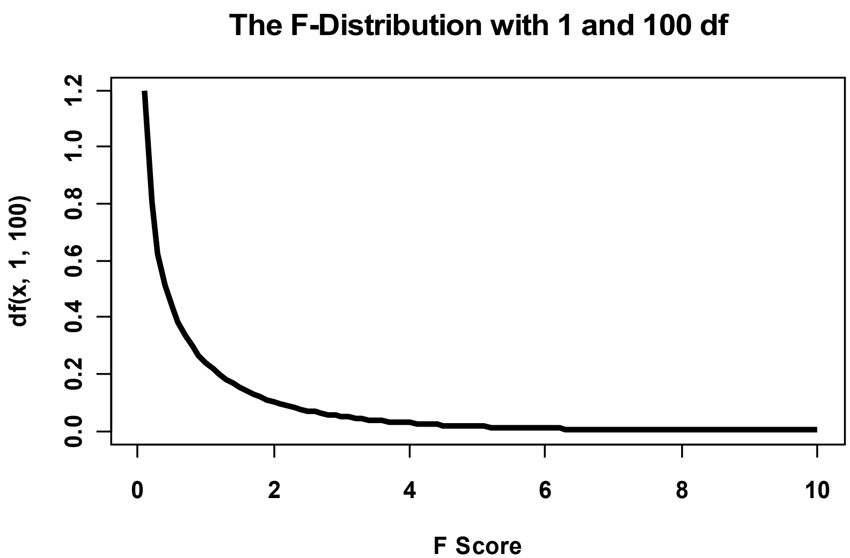
ANOVA: The *F* Statistic

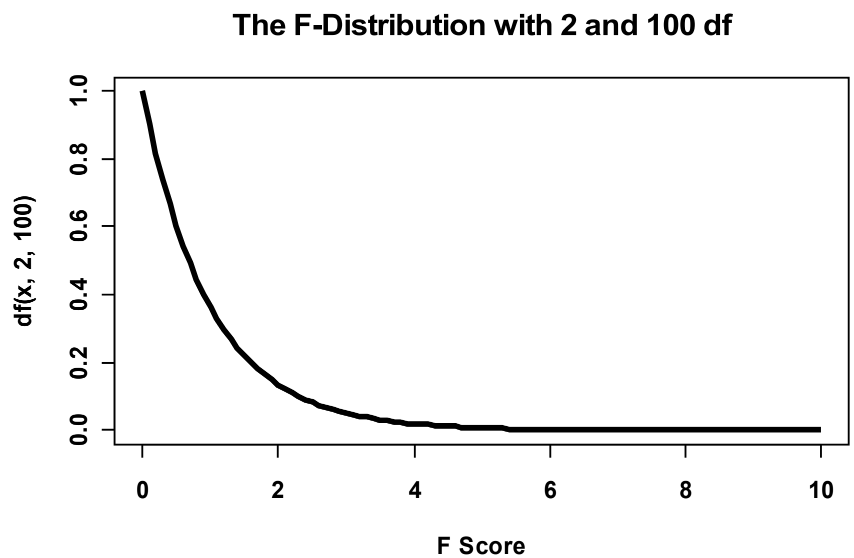
The final ANOVA (*F*) statistic is

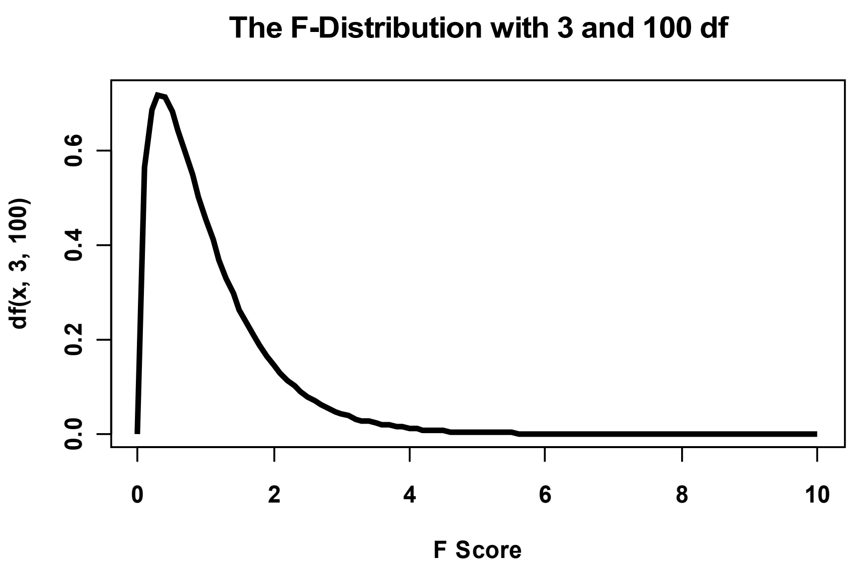
$$F(df_B, df_W) = \frac{MSBetween}{MSWithin}$$

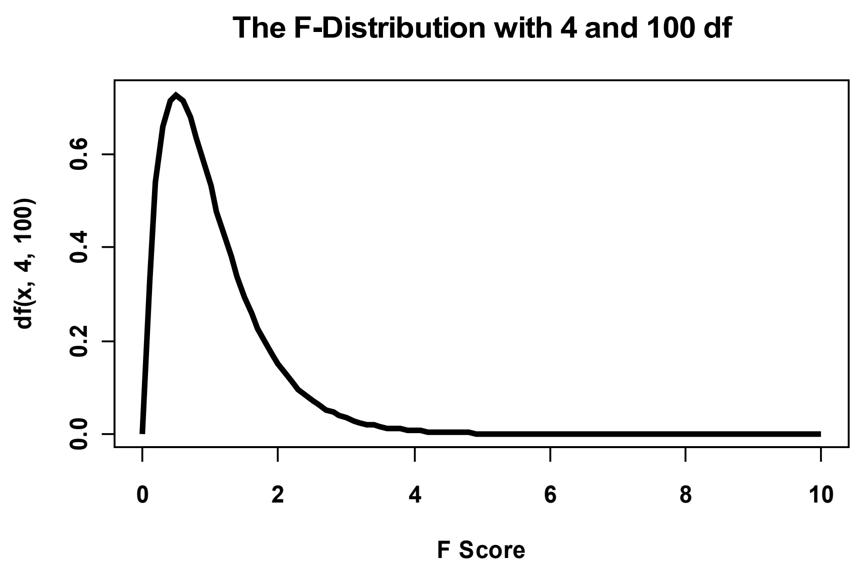
where df_B and df_W are the degrees of freedom for the between and within groups and

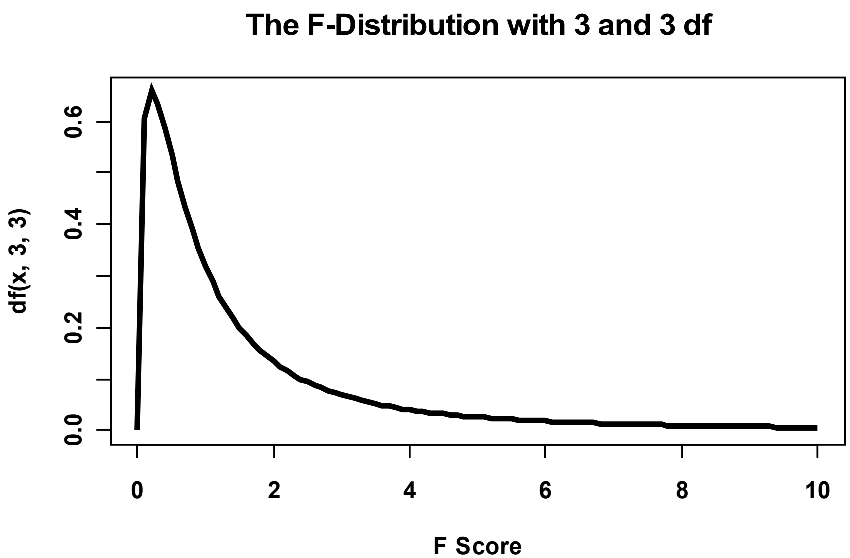
MS is the mean square for the appropriate group.

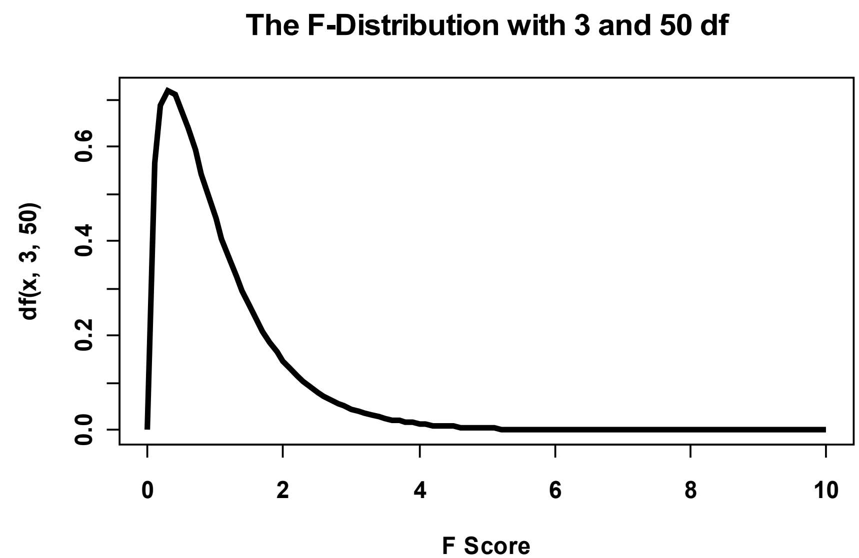

The F-Distribution with 1 and 100 df
df(x, 1, 100)
0.0
0.2
0.4
0.6
0.8
1.0
1.2
0
2
4
6
8
10
F Score

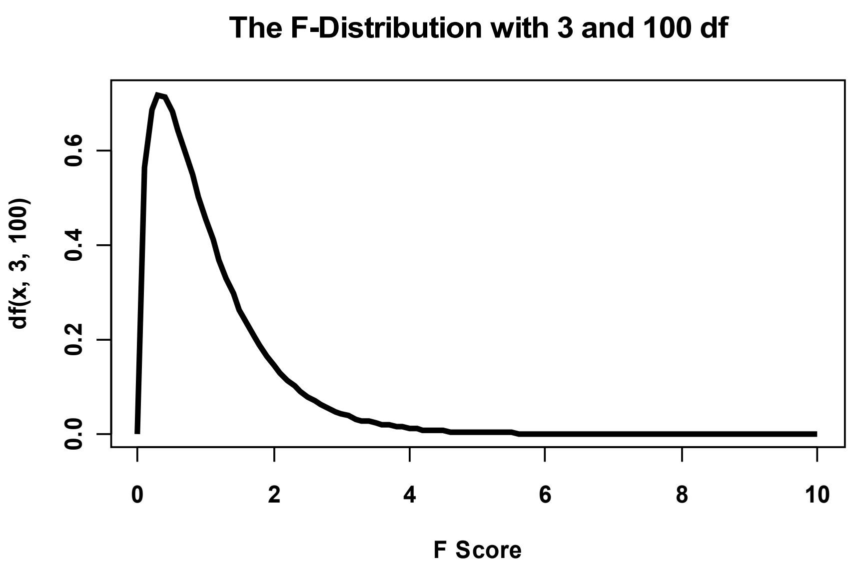

The F-Distribution with 2 and 100 df
df(x, 2, 100)
0.0
0.2
0.4
0.6
0.8
1.0
0
2
4
6
8
10
F Score

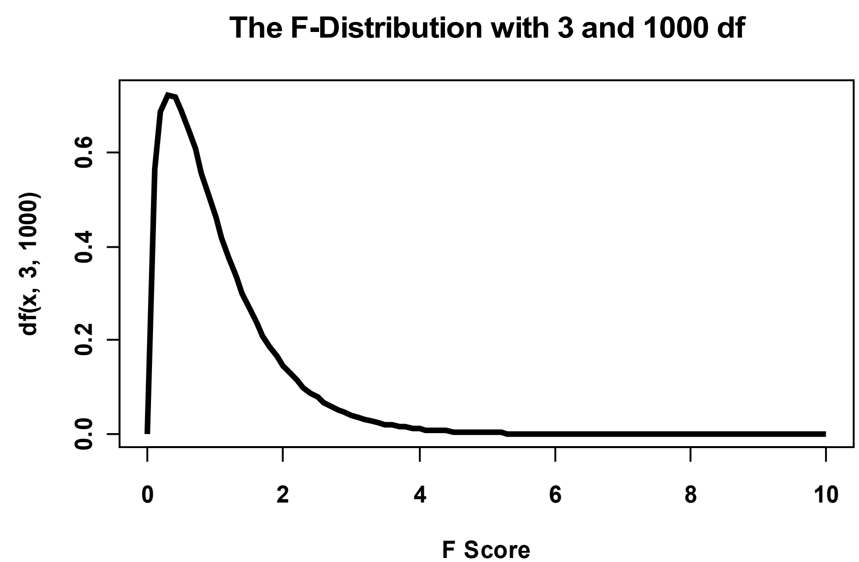

The F-Distribution with 3 and 100 df
df(x, 3, 100)
0.0
0.2
0.4
0.6
0
2
4
6
8
10
F Score

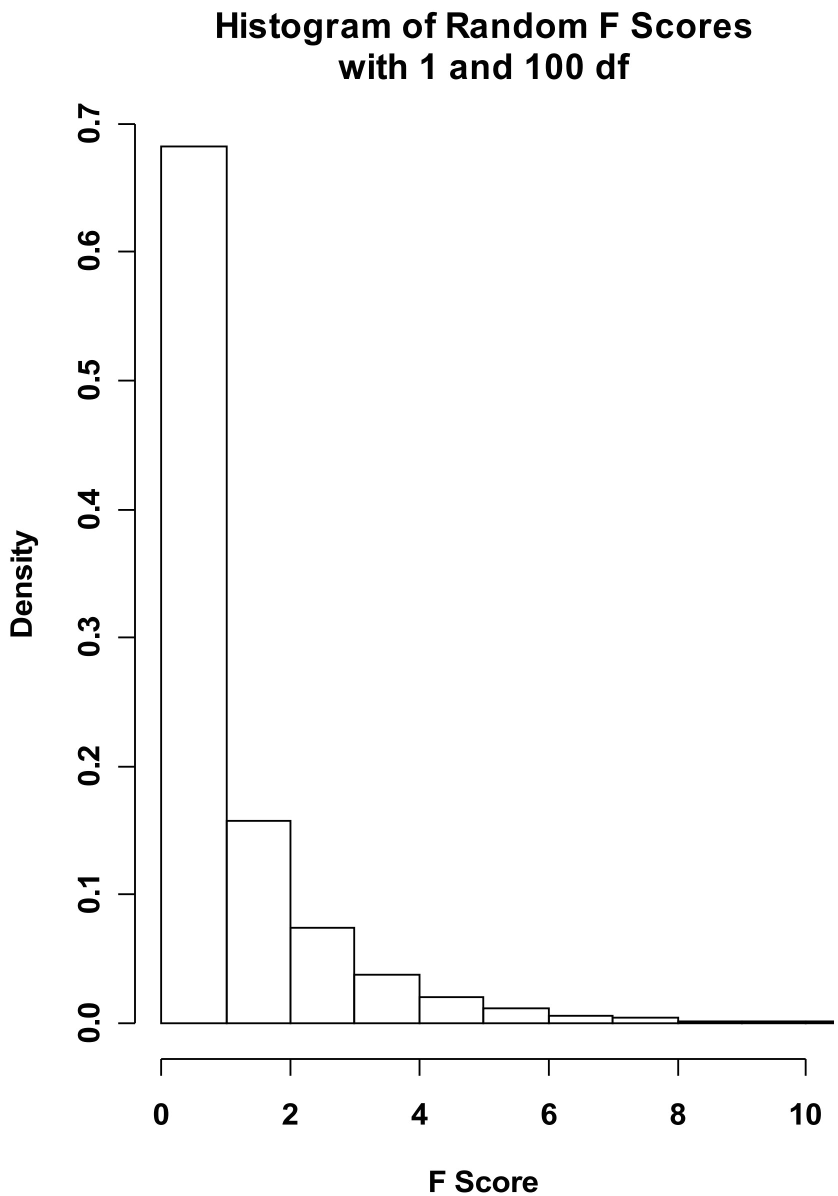

The F-Distribution with 4 and 100 df
df(x, 4, 100)
0.0
0.2
0.4
0.6
0
2
4
6
8
10
F Score

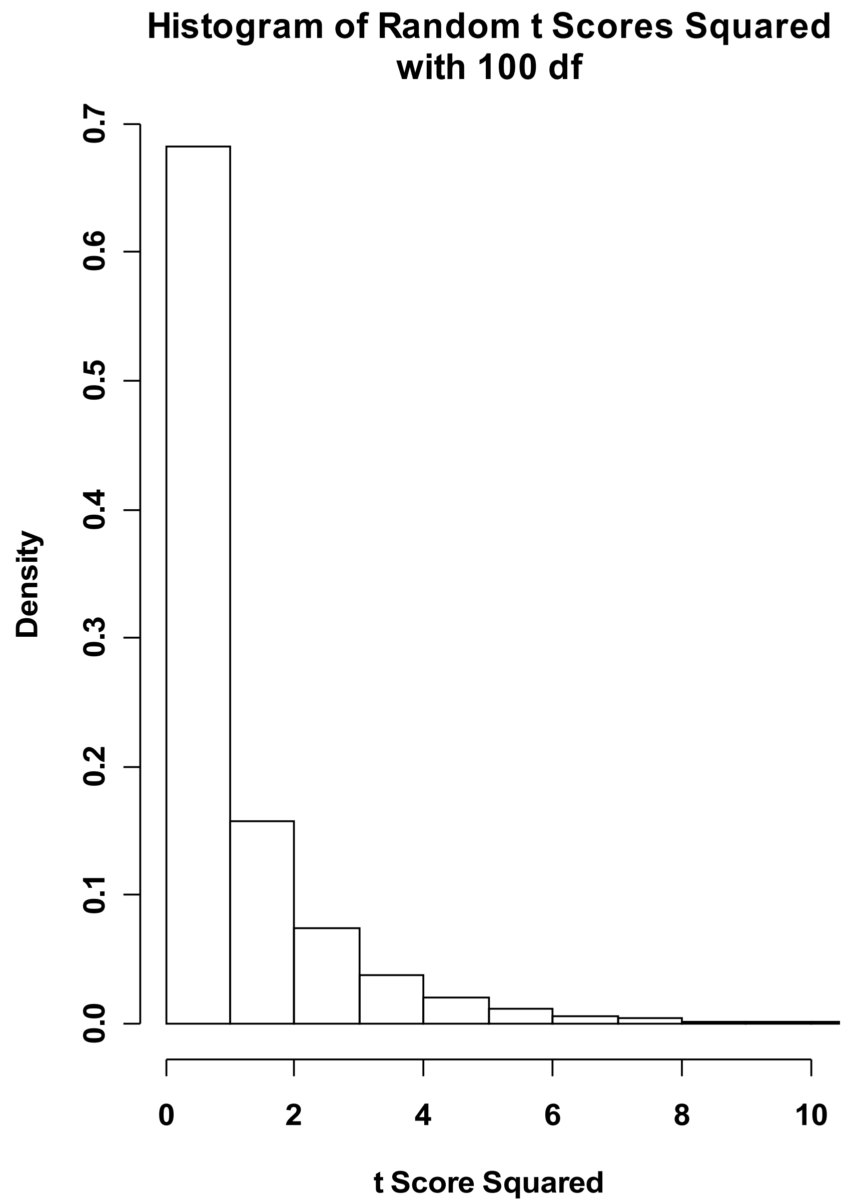

The F-Distribution with 3 and 3 df
df(x, 3, 3)
0.0
0.2
0.4
0.6
0
2
4
6
8
10
F Score


The F-Distribution with 3 and 50 df
df(x, 3, 50)
0.0
0.2
0.4
0.6
0
2
4
6
8
10
F Score


The F-Distribution with 3 and 100 df
df(x, 3, 100)
0.0
0.2
0.4
0.6
0
2
4
6
8
10
F Score


The F-Distribution with 3 and 1000 df
df(x, 3, 1000)
0.0
0.2
0.4
0.6
0
2
4
6
8
10
F Score


Histogram of Random F Scores
with 1 and 100 df
Density
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0
2
4
6
8
10
F Score


Histogram of Random t Scores Squared
with 100 df
Density
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0
2
4
6
8
10
t Score Squared

ANOVA: The Calculation

Look at Howell, p. 385 for an example. Based on the information provided in my incomplete table, you should be able to fill out the entire ANOVA table.

Source	Sum of Sq.	df	Mean Sq.	F
Between	351.52	4	x	x
Within	x	x	x	
Total	786.82	49		

What does this tell us?

ANOVA: The Assumptions

The assumptions of ANOVA are:

Normality of the populations.

Homogeneity of variance.

Independence of observations.

Do these sound familiar?

Keep in mind that ___________________ is just a specific case of the one-way ANOVA.

Multiple (Post-Hoc) Comparisons

The overall ANOVA tells us whether or not there is a difference. It does not tell us ________________________. Multiple comparisons (Post-Hoc tests) point to the specific difference.

Fisher's LSD – Breaks down the mean differences and MSWithin into a group of special *t*-tests. Very liberal in the α error sense (more likely to find a difference).

Bonferroni – Breaks down groups into a different type of *t*-test. The *t*-test critical α values are adjusted by the number of tests run. Uber conservative in the α error sense.

Tukey's HSD – Finds the smallest mean difference that is "honestly significant". Slightly conservative.

How Big is the Effect?

This is easy!

The statistic: η^2 (eta squared)

$$\eta^2 = \frac{SSBetween}{SSTotal}$$

The percentage of variance accounted for by group membership!
H, 403

Remember (and use) this statistic!!!!!!!

ANOVA with More Than 1 IV

You can do an analysis of variance with 2, 3, or more independent variables. This is called Factorial ANOVA

With 1 variable:

Drug 1	Drug 2	Placebo	Total
Mean 1 (Var)	Mean 2 (Var)	Mean 3 (Var)	Gr. Mean (Var)

With 2 variables (simplest case):

	Drug	No Drug
Counseling	Mean (Var)	Mean (Var)
No Counseling	Mean (Var)	Mean (Var)

Factorial ANOVA: Why It is Difficult?

Only 1 IV is easy. If there is a significant effect, it MUST be the IV.

With more than 1 IV, people are getting different combinations of IVs. We must deal with the ____________ between levels of the IV.

Main effect: Is there a significant mean difference when looking only at ____________________?

Interaction: Is there a significant mean difference when looking at the variables acting together? The effect goes above and beyond the individual IVs by themselves.

ANOVA: 2 x 2 Example

Consider the following means. DV = Depression score

	Drug	No Drug	Means
Counseling	58	68	
No Counseling	60	70	
Means			

Knowing nothing about significance yet, it might be nifty to plot the means. Let's plot this on the board.

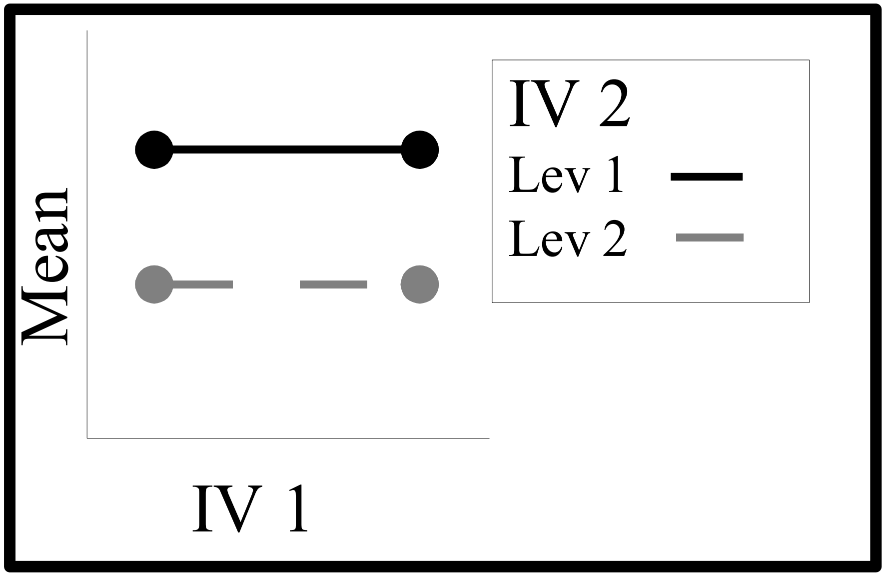
ANOVA: 2 x 2 Interaction Plots

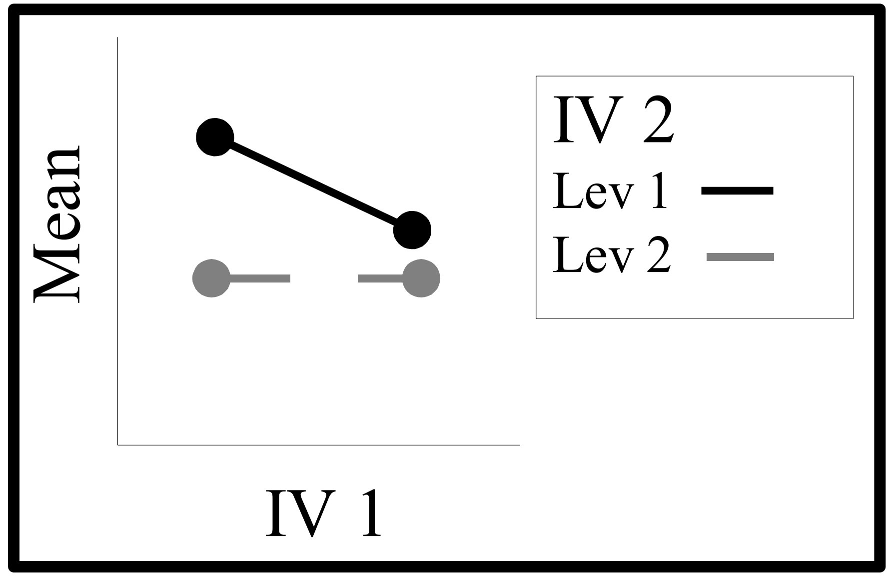
Even if you do not know what the factorial ANOVA statistics mean yet, you can interpret graphs of means.

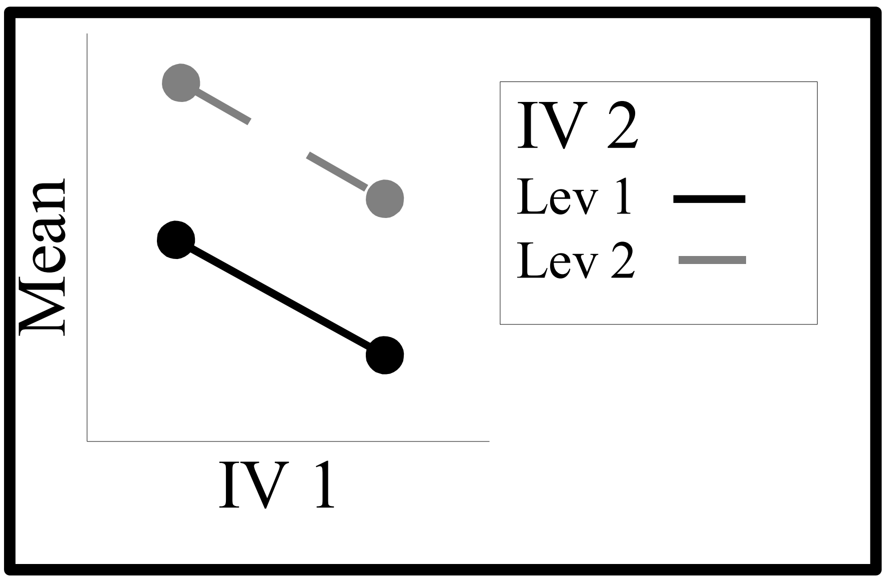
Interaction plot: A plot of cell means with one IV represented on the X-axis and another IV represented by different lines.

Main effect: There is a difference in the levels of the lines and/or the points in the graph. Look at the halfway point.

Interaction: The lines are not (close to) parallel.


IV 2
Lev 1
Lev 2
Mean
IV 1


IV 2
Lev 1
Lev 2
Mean
IV 1


IV 2
Lev 1
Lev 2
Mean
IV 1