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# THE DECOMPOSITION OF EFFECTS IN PATH ANALYSIS\*

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*This paper is about the logic of interpreting recursive causal theories in sociology. We review the distinction between associations and effects and discuss the decomposition of effects into direct and indirect components. We then describe a general method for decomposing effects into their components by the systematic application of ordinary least squares regression. The method involves successive computation of reduced-form equations, beginning with an equation containing only exogenous variables, then computing equations which add intervening variables in sequence from cause to effect. This generates all the information required to decompose effects into their various direct and indirect parts. This method is a substitute for the often more cumbersome computation of indirect effects from the structural coefficients (direct effects) of the causal model. Finally, we present a way of summarizing this information in tabular form and illustrate the procedures using an empirical example.*

## INTRODUCTION

The construction of linear recursive models using path analysis and multiple regression has become widely recognized as a useful approach to quantifying and interpreting causal theory in sociology (e.g., Goldberger and Duncan, 1973; Anderson and Evans, 1974; Lewis-Beck, 1974). The diffusion of causal modeling among sociologists is primarily due to the work of Blalock (1961, 1969) and Duncan (1966), and secondarily to papers by Boudon (1965), Land (1969), and Heise (1969). Recent discussions of path analysis (Duncan, 1971; Finney, 1972) have suggested corrections in the language used to describe the components of statistical relationships among variables. Of primary importance is the distinction between *total effects* and *total associations*, not observed in earlier

treatments of path analysis (Duncan, 1966; Land, 1969). A related issue is the definition of *indirect effects*.

This paper will describe a general method for decomposing total effects into their constituent direct and indirect effects. The procedure builds on an interpretive scheme applied elsewhere (Hauser et al., 1975; Alwin et al., 1975), which involves the systematic use of reduced-form equations. We shall demonstrate how estimating successive reduced-form equations in the interpretation of recursive path models substitutes for the more cumbersome computation of direct and indirect effects from the coefficients of each structural equation in a recursive model. Our aims are strictly didactic. We make no claims of originality, and we are heavily indebted to Duncan, Featherman and Duncan's (1972:Ch. 2) presentation of similar material. Our discussion assumes a working knowledge of the rules and notational conventions of path analysis and of the language of multiple regression analysis. However, this is not a paper about statistics or regression analysis. Rather, our purpose is to exposit methods for interpreting one type of sociological theory. We begin our discussion by recapitulating the important distinction between total associations and total effects.

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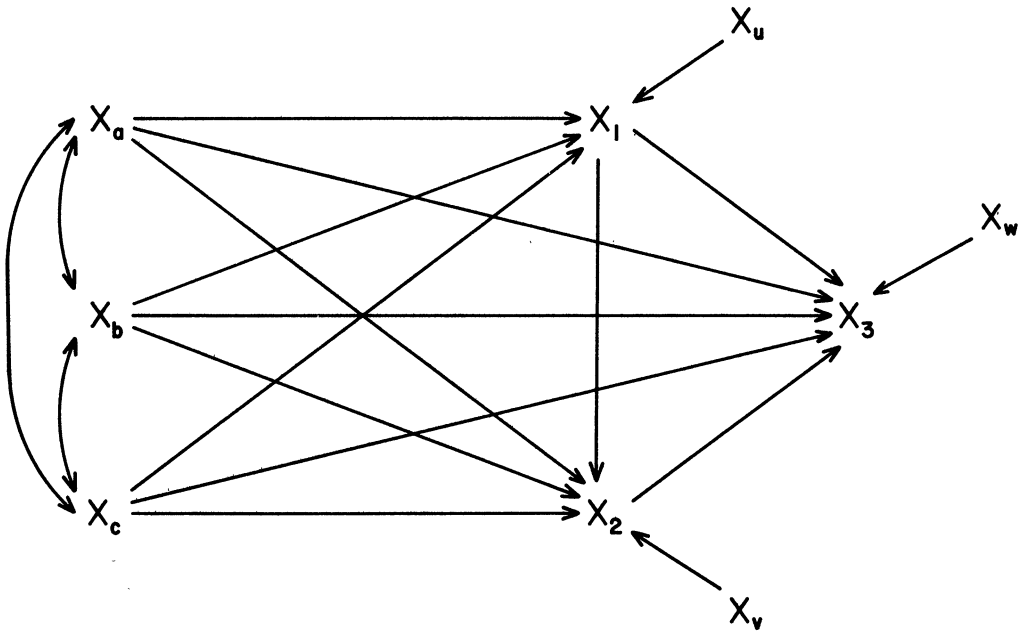


Fig. 1 Causal Diagram For A Linear Recursive Model

#### Associations Versus Effects

Consider the recursive model depicted in the causal diagram in Figure 1 and represented by the following set of structural equations.

$$X_1 = p_{1a} X_a + p_{1b} X_b + p_{1c} X_c + p_{1u} X_u \quad (1)$$

$$X_2 = p_{2a} X_a + p_{2b} X_b + p_{2c} X_c + p_{21} X_1 + p_{2v} X_v \quad (2)$$

$$X_3 = p_{3a} X_a + p_{3b} X_b + p_{3c} X_c + p_{31} X_1 + p_{32} X_2 + p_{3w} X_w \quad (3)$$

We assume the observed variables in the model ( $X_a, X_b, X_c, X_1, X_2,$  and  $X_3$ ) are measured on interval scales without error. The random disturbances ( $X_u, X_v, X_w$ ) are mutually uncorrelated and, also, uncorrelated with observed variables on the right-hand side of the structural equation in which they appear, i.e.,  $\rho_{au} = \rho_{bu} = \rho_{cu} = \rho_{av} = \rho_{bv} = \rho_{cv} = \rho_{1v} = \rho_{aw} = \rho_{bw} = \rho_{cw} = \rho_{1w} = \rho_{2w} = \rho_{uw} = \rho_{vw} = 0$ .<sup>1</sup> Without loss of generality we assume all variables are normal-

ized with mean of zero and standard deviation of unity.

The model is stated for the population, but in ordinary least-squares regression analysis our results will hold for corresponding sample quantities. Consistent with the notation of path analysis, we symbolize direct effects by  $p$ 's, e.g.,  $p_{1a}$  is the direct effect of  $X_a$  on  $X_1$ . Following Duncan, Featherman and Duncan (1972) we shall symbolize total effects by  $q$ 's, e.g.,  $q_{2a}$  is the total effect of  $X_a$  on  $X_2$ . Sometimes a total effect is simply equal to the direct effect, e.g.,  $q_{1a} = p_{1a}$ , and in such cases the  $p$ -notation is used.

We first give verbal definitions of the causal and noncausal components of association between variables and then elaborate them with reference to Figure 1. Our discussion applies where the *total association* between two variables is given by their zero-order correlation. The *total effect* of one variable on another is the part of their total association which is neither due to their

<sup>1</sup> Given that disturbances are uncorrelated with regressors in each equation, the assumption of mutually uncorrelated disturbances is redundant.

common causes, to correlation among their causes, nor to unanalyzed (predetermined) correlation (Duncan, 1971). In common usage the first two of these noncausal components of association are called "spurious," but there is no convenient term for the last. A total effect tells us how much change in a consequent variable is induced by a given shift in an antecedent variable, irrespective of the mechanisms by which the change may occur. Of course the validity of a total effect is always conditioned on the correct specification of noncausal components of association.

*Indirect effects* are those parts of a variable's total effect which are transmitted or mediated by variables specified as intervening between the cause and effect of interest in a model. That is, they tell us how much of a given effect occurs because the manipulation of the antecedent variable of interest leads to changes in other variables which in turn change the consequent variable. The *direct effect* of one variable on another is simply that part of its total effect which is *not* transmitted via intervening variables. It is the effect which remains when intervening variables have been held constant. Although we refer to unmediated effects as "direct," the possibility always exists that additional intervening variables may transmit part or all of the effect. In using causal terms it is necessary to specify the model involved. The distinction between direct and indirect effects, like that between causal and noncausal components of association, relates to a specific causal model.

We can illustrate the definitions just given by applying the basic theorem of path analysis (Duncan, 1966) to selected correlations between variables in Figure 1. The basic theorem expresses correlations among variables in terms of parameters of the model according to the rule

$$\rho_{ij} = \sum_k p_{ik} \rho_{kj}, \quad (4)$$

where  $j$  indexes the causal variable of interest,  $i$  indexes the affected variable, and the index  $k$  varies over all variables from which paths lead directly to variable  $i$ . For example, we may write  $\rho_{1a}$  as

$$\rho_{1a} = p_{1a} + p_{1b} \rho_{ba} + p_{1c} \rho_{ca}. \quad (5)$$

Here,  $\rho_{1a}$  is the total association between  $X_a$  and

$X_1$ ,  $p_{1a}$  is both the total and direct effect of  $X_a$  on  $X_1$ , and  $p_{1b} \rho_{ba}$  and  $p_{1c} \rho_{ca}$  are components of association due to the correlation of  $X_a$  with the other two measured causes of  $X_1$ . (In the special cases where  $\rho_{ab} = \rho_{ac} = 0$ , or where  $X_a$  is the only measured cause of  $X_1$ , then  $p_{1a} = \rho_{1a}$  is the direct effect, total effect, and total association.) For  $\rho_{2a}$  we may write

$$\rho_{2a} = p_{2a} + p_{2b} \rho_{ba} + p_{2c} \rho_{ca} + p_{21} \rho_{1a}, \quad (6)$$

and substituting equation 5 for  $\rho_{1a}$  we have

$$\begin{aligned} \rho_{2a} &= p_{2a} + p_{2b} \rho_{ba} + p_{2c} \rho_{ca} + p_{21} (p_{1a} \\ &\quad + p_{1b} \rho_{ba} + p_{1c} \rho_{ca}) \\ &= p_{2a} + p_{21} p_{1a} + p_{2b} \rho_{ba} + p_{2c} \rho_{ca} \\ &\quad + p_{21} p_{1b} \rho_{ba} + p_{21} p_{1c} \rho_{ca}. \end{aligned} \quad (7)$$

From equation 7 we see that the model gives six components of the total association between  $X_a$  and  $X_2$ . The total effect of  $X_a$  on  $X_2$  is given by the sum of the direct effect, ( $p_{2a}$ ) and the indirect effect via  $X_1$  ( $p_{21} p_{1a}$ ):

$$q_{2a} = p_{2a} + p_{21} p_{1a}. \quad (8)$$

The remaining association between  $X_a$  and  $X_2$  is attributable to the correlation of  $X_a$  with each of  $X_b$  and  $X_c$  and their respective effects on  $X_2$ , both direct ( $p_{2b} \rho_{ba}$  and  $p_{2c} \rho_{ca}$ ) and indirect ( $p_{21} p_{1b} \rho_{ba}$  and  $p_{21} p_{1c} \rho_{ca}$ ). This interpretation of the noncausal components of  $\rho_{2a}$  may be easier to follow if we note equation 7 may be written as

$$\rho_{2a} = p_{2a} + p_{21} p_{1a} + (p_{2b} + p_{21} p_{1b}) \rho_{ba} + (p_{2c} + p_{21} p_{1c}) \rho_{ca}. \quad (9)$$

To give an example of spurious components of association we write an expression for the association between the two endogenous variables,  $X_1$  and  $X_2$ .

$$\begin{aligned} \rho_{21} &= p_{2a} \rho_{a1} + p_{2b} \rho_{b1} + p_{2c} \rho_{c1} + p_{21} \\ &= p_{2a} (p_{1a} + p_{1b} \rho_{ba} + p_{1c} \rho_{ca}) \\ &\quad + p_{2b} (p_{1a} \rho_{ab} + p_{1b} + p_{1c} \rho_{cb}) \\ &\quad + p_{2c} (p_{1a} \rho_{ac} + p_{1b} \rho_{bc} + p_{1c}) + p_{21} \\ &= p_{21} + p_{2a} p_{1a} + p_{2b} p_{1b} + p_{2c} p_{1c} \\ &\quad + p_{2a} p_{1b} \rho_{ba} + p_{2a} p_{1c} \rho_{ca} \\ &\quad + p_{2b} p_{1a} \rho_{ab} \\ &\quad + p_{2b} p_{1c} \rho_{cb} + p_{2c} p_{1a} \rho_{ac} \\ &\quad + p_{2c} p_{1b} \rho_{bc}. \end{aligned} \quad (10)$$

Equation 10 shows  $p_{21}$  is the only causal component of the association between  $X_2$  and  $X_1$ . Thus, it is both a total and a direct effect. The other nine components of  $\rho_{21}$  all represent spurious correlation. Three express the common dependence of  $X_2$  and  $X_1$  on  $X_a$ ,  $X_b$  and  $X_c$  ( $p_{2a}p_{1a}$ ,  $p_{2b}p_{1b}$ , and  $p_{2c}p_{1c}$ , respectively), and the other six terms involve unanalyzed correlations between measured causes of  $X_1$  and  $X_2$ . Specifically, they express the dependence of  $X_1$  and  $X_2$  on correlated causes.

Summarizing the discussion up to this point, the correlation (or association) between any two variables in a recursive model contains one or more of the following components: correlation involving unanalyzed association among predetermined variables, correlation due to joint dependence on common or correlated variables, and effects. The last may in turn be expressed as the sum of direct and indirect effects. As we have already hinted (see equation 9), there are numerous combinations of these components of association, but we think further enumeration of them is unnecessary. Up to this point we have merely reiterated points made by Duncan (1971) and Finney (1972). Our main concern is the interpretation of total effects in recursive models. The following discussion presents a method for decomposing total effects into their direct and indirect parts by applying multiple regression procedures systematically.

### *Interpretation of Effects*

Users of path analysis know the calculation of indirect and total effects from even a moderately complicated path diagram is tedious and cumbersome. It invites errors of logic and errors in computation and rounding. We suspect these difficulties have led some researchers to avoid a full interpretation of their findings even where it would speak directly to their theoretical interests. However, given an accurate and efficient computer program for ordinary least-squares regression, the total effects of variables in recursive models can easily be obtained without summing the products of direct effects. Also, by estimating a series of regression equations, it is possible to ascertain indirect effects without any arithmetic skills beyond addition

and subtraction, once the computer has done its work.

Suppose we substitute equation 1 for  $X_1$  in equation 2. We have

$$\begin{aligned} X_2 &= p_{2a}X_a + p_{2b}X_b + p_{2c}X_c \\ &\quad + p_{21}(p_{1a}X_a + p_{1b}X_b + p_{1c}X_c + p_{1u}X_u) \\ &\quad + p_{2v}X_v \\ &= (p_{2a} + p_{21}p_{1a})X_a + (p_{2b} + p_{21}p_{1b})X_b \\ &\quad + (p_{2c} + p_{21}p_{1c})X_c + p_{21}p_{1u}X_u \\ &\quad + p_{2v}X_v \end{aligned} \quad (11)$$

The first term on the right-hand side in parentheses is just what we had earlier defined as  $q_{2a}$  (see equation 8), the total effect of  $X_a$  on  $X_2$ . Adopting similar notation for the total effects of  $X_b$  and  $X_c$  on  $X_2$ , we may rewrite equation 11 as

$$\begin{aligned} X_2 &= q_{2a}X_a + q_{2b}X_b + q_{2c}X_c + p_{21}p_{1u}X_u \\ &\quad + p_{2v}X_v \end{aligned} \quad (12)$$

Equation 12 expresses  $X_2$  as a linear function of the total effects of  $X_a$ ,  $X_b$ , and  $X_c$  and of the disturbance variables ( $X_u, X_v$ ). We assumed initially that  $X_u$  and  $X_v$  were uncorrelated with  $X_a$ ,  $X_b$ , or  $X_c$ , and by virtue of that assumption we know that  $q_{2a}$ ,  $q_{2b}$ , and  $q_{2c}$  are the same coefficients we would obtain directly by regressing  $X_2$  on  $X_a$ ,  $X_b$ , and  $X_c$ . That is, suppose we first estimate the reduced-form equation,<sup>2</sup>

$$\begin{aligned} X_2 &= q_{2a}X_a + q_{2b}X_b + q_{2c}X_c \\ &\quad + q'_{2v}X'_v \end{aligned} \quad (13)$$

(where  $q'_{2v}X'_v = p_{21}p_{1u}X_u + p_{2v}X_v$  by definition), and then we estimate the structural equation which incorporates  $X_1$  as a regressor (equation 2). The coefficients of  $X_a$ ,  $X_b$ , and  $X_c$  in the reduced-form equation 13 —  $q_{2a}$ ,  $q_{2b}$ , and  $q_{2c}$ , respectively — are the

<sup>2</sup>Following Duncan, Featherman and Duncan (1972:23-30) we apply the term "reduced-form" to any equation in which none of the antecedents of any regressor have been omitted. Note that reduced-form equations are to be distinguished from semi-reduced-form equations. See Johnston (1972) for further discussion of reduced-form equations.

total effects of those three variables. The coefficients of  $X_a$ ,  $X_b$ , and  $X_c$  in the structural equation 2 —  $p_{2a}$ ,  $p_{2b}$ , and  $p_{2c}$ , respectively — are the direct effects of those three variables. The differences between the respective coefficients of  $X_a$ ,  $X_b$ , and  $X_c$  in equations 2 and 13 are the indirect effects of those three variables,

$$q_{2a} - p_{2a} = p_{21}p_{1a} \quad (14)$$

$$q_{2b} - p_{2b} = p_{21}p_{1b} \quad (15)$$

$$q_{2c} - p_{2c} = p_{21}p_{1c}, \quad (16)$$

which we obtain *without* regressing  $X_1$  on  $X_a$ ,  $X_b$ , and  $X_c$ .

As an aside we note that the proportionate decomposition of an effect into its direct and indirect components is invariant with respect to the standardization of variables. Suppose we had carried out the regressions of equations 2 and 13 for variables in their original metric, rather than variables in standard form. For example, instead of equation 14, we would have

$$\beta_{2a.bc} - \beta_{2a.1bc} = \beta_{21.abc} \beta_{1a.bc}, \quad (17)$$

where  $\beta_{ij.kl}$  is the regression coefficient of  $X_i$  on  $X_j$  net of variables  $k$  and  $l$  in the population. But by definition a regression coefficient for variables in standard form is the product of a regression coefficient and the ratio of the standard deviations of pre-determined and dependent variables, e.g.,  $q_{2a} = \beta_{2a.bc} \sigma_a / \sigma_2$ . Thus, we may rewrite equation 17 as

$$q_{2a}(\sigma_2 / \sigma_a) - p_{2a}(\sigma_2 / \sigma_a) = p_{21}(\sigma_2 / \sigma_1) p_{1a}(\sigma_1 / \sigma_a), \quad (18)$$

from which it is evident by inspection that equations 17 and 14 differ only by the constant of proportionality,  $\sigma_2 / \sigma_a$ . One important consequence of this result is that it is sometimes possible to make interpopulation comparisons of the proportionate decomposition of an effect into its direct and indirect components even where the variables of interest have not been measured in the same metric in each population.

The general idea that direct and indirect effects can be obtained from the coefficients of reduced-form and structural equations extends to models with more than one intervening variable. For example, suppose we substitute equation 2 for  $X_2$  in equation 3. We have

$$\begin{aligned} X_3 &= p_{3a}X_a + p_{3b}X_b + p_{3c}X_c + p_{31}X_1 \\ &\quad + p_{32}(p_{2a}X_a + p_{2b}X_b + p_{2c}X_c \\ &\quad + p_{21}X_1 + p_{2v}X_v) + p_{3w}X_w \\ &= (p_{3a} + p_{32}p_{2a})X_a + (p_{3b} + p_{32}p_{2b})X_b \\ &\quad + (p_{3c} + p_{32}p_{2c})X_c + (p_{31} \\ &\quad + p_{32}p_{21})X_1 + p_{32}p_{2v}X_v \\ &\quad + p_{3w}X_w. \end{aligned} \quad (19)$$

Defining the notation

$$q_{3a}^* = p_{3a} + p_{32}p_{2a} \quad (20)$$

$$q_{3b}^* = p_{3b} + p_{32}p_{2b} \quad (21)$$

$$q_{3c}^* = p_{3c} + p_{32}p_{2c} \quad (22)$$

$$q_{31} = p_{31} + p_{32}p_{21} \quad (23)$$

$$\text{and } p'_{3w}X'_w = p_{32}p_{2v}X_v + p_{3w}X_w,$$

we may rewrite equation 19 as

$$\begin{aligned} X_3 &= q_{3a}^*X_a + q_{3b}^*X_b + q_{3c}^*X_c + q_{31}X_1 \\ &\quad + p'_{3w}X'_w. \end{aligned} \quad (24)$$

As before, we have assumed  $X_v$  and  $X_w$  are uncorrelated with  $X_a$ ,  $X_b$ ,  $X_c$ , or  $X_1$ , so we may obtain  $q_{3a}^*$ ,  $q_{3b}^*$ ,  $q_{3c}^*$ , and  $q_{31}$  directly by regressing  $X_3$  on  $X_a$ ,  $X_b$ ,  $X_c$ , and  $X_1$ . It should be clear from Figure 1 that  $q_{31}$  is the total effect of  $X_1$  on  $X_3$ , since  $X_1$  may only affect  $X_3$  directly or by way of  $X_2$ . Thus, we may interpret the influence of  $X_3$  on  $X_1$  by obtaining the total effect,  $q_{31}$ , from the reduced-form equation 24, obtaining the direct effect,  $p_{31}$ , from equation 3, and obtaining the indirect effect of  $X_1$  via  $X_2$  by  $q_{31} - p_{31} = p_{32}p_{21}$ .

While  $q_{31}$  in equation 24 is a total effect,  $q_{3a}^*$ ,  $q_{3b}^*$ , and  $q_{3c}^*$  are not the respective total effects of  $X_a$ ,  $X_b$ , and  $X_c$ .

Like  $q_{31}$ , the reduced-form coefficients,  $q_{3a}^*$ ,  $q_{3b}^*$ , and  $q_{3c}^*$ , include the direct effects of  $X_a$ ,  $X_b$ , and  $X_c$ , respectively, and their indirect effects via  $X_2$ ; but  $q_{3a}^*$ ,  $q_{3b}^*$ , and  $q_{3c}^*$  are not total effects because they exclude the indirect effects of  $X_a$ ,  $X_b$ , and  $X_c$  by way of  $X_1$ , and by way of both  $X_1$  and  $X_2$ . For example, we may obtain the effect  $q_{3a}^*$  from equation 24 and  $p_{3a}$  from equation 3; and their difference,  $q_{3a}^* - p_{3a} = p_{32}p_{2a}$ , gives the indirect effect of  $X_a$  on  $X_3$  by way of  $X_2$ . However, none of these results sheds any light on the indirect effects of  $X_a$  on  $X_3$  by way of  $X_1$  alone ( $p_{31}p_{1a}$ ) or by way of  $X_1$  and  $X_2$  ( $p_{32}p_{21}p_{1a}$ ).

To complete the interpretation of the effects of  $X_a$ ,  $X_b$ , and  $X_c$  on  $X_3$ , we can substitute equation 1 for  $X_1$  in equation 24. We have

$$\begin{aligned} X_3 &= q_{3a}^* X_a + q_{3b}^* X_b + q_{3c}^* X_c + q_{31} (p_{1a} X_a \\ &\quad + p_{1b} X_b + p_{1c} X_c + p_{1u} X_u) + p'_{3w} X'_w \\ &= (q_{3a}^* + q_{31} p_{1a}) X_a + (q_{3b}^* + q_{31} p_{1b}) X_b \\ &\quad + (q_{3c}^* + q_{31} p_{1c}) X_c + q_{31} p_{1u} X_u + p'_{3w} X'_w \\ &= q_{3a} X_a + q_{3b} X_b + q_{3c} X_c + p''_{3w} X''_w, \end{aligned} \quad (25)$$

$$\text{where } q_{3a} = q_{3a}^* + q_{31} p_{1a} = p_{3a} + p_{32} p_{2a} + (p_{31} + p_{32} p_{21}) p_{1a}, \quad (26)$$

$$\begin{aligned} q_{3b} &= q_{3b}^* + q_{31} p_{1b} = p_{3b} + p_{32} p_{2b} \\ &\quad + (p_{31} + p_{32} p_{21}) p_{1b}, \end{aligned} \quad (27)$$

$$\text{and } q_{3c} = q_{3c}^* + q_{31} p_{1c} = p_{3c} + p_{32} p_{2c} + (p_{31} + p_{32} p_{21}) p_{1c}, \quad (28)$$

are the total effects on  $X_3$  of  $X_a$ ,  $X_b$ , and  $X_c$ , respectively, and

$$\begin{aligned} p''_{3w} X''_w &= q_{31} p_{1u} X_u + p'_{3w} X'_w \\ &= q_{31} p_{1u} X_u + p_{32} p_{2v} X_v \\ &\quad + p_{3w} X_w \end{aligned} \quad (29)$$

Again, by virtue of our initial assumptions the total effects  $q_{3a}$ ,  $q_{3b}$ , and  $q_{3c}$  may be obtained directly by regressing  $X_3$  on  $X_a$ ,  $X_b$ , and  $X_c$ .

To obtain a complete interpretation of the effects of  $X_a$ ,  $X_b$ , and  $X_c$  on  $X_3$  in the model

of Figure 1, we first regress  $X_3$  on  $X_a$ ,  $X_b$ , and  $X_c$  to obtain  $q_{3a}$ ,  $q_{3b}$ , and  $q_{3c}$ . Next, we regress  $X_3$  on  $X_a$ ,  $X_b$ ,  $X_c$ , and  $X_1$  to obtain  $q_{3a}^*$ ,  $q_{3b}^*$ ,  $q_{3c}^*$ , and  $q_{31}$ . Then, we regress  $X_3$  on  $X_a$ ,  $X_b$ ,  $X_c$ ,  $X_1$  and  $X_2$  to obtain  $p_{3a}$ ,  $p_{3b}$ ,  $p_{3c}$ ,  $p_{31}$ , and  $p_{32}$ . We can interpret the total effects ( $q_{ij}$ ) by taking differences between coefficients of each variable in successive equations. For example,  $q_{3a}$  is the total effect of  $X_a$  on  $X_3$ . Next,  $q_{3a} - q_{3a}^* = q_{31} p_{1a} = (p_{31} + p_{32} p_{21}) p_{1a}$  is the indirect effect of  $X_a$  on  $X_3$  by way of  $X_1$  and its direct and indirect effects on  $X_3$ . Next,  $q_{3a}^* - p_{3a} = p_{32} p_{2a}$  is the indirect effect of  $X_a$  on  $X_3$  by way of  $X_2$  and its direct (and, possibly, indirect) effects on  $X_3$ . Finally,  $p_{3a}$  is the direct effect of  $X_a$  on  $X_3$ ; but this is subject to further interpretation if other causes intervene between  $X_2$  and  $X_3$ . Note the difference  $q_{3a} - q_{3a}^*$  includes two compound paths ( $p_{31} p_{1a}$  and  $p_{32} p_{21} p_{1a}$ ), but no information is thereby lost, for the components of the effect of  $X_1$  on  $X_3$  directly and by way of  $X_2$  are necessarily revealed by the comparison of  $q_{31}$  and  $p_{31}$ .

In summary, we have developed a method for interpreting the effects of variables in recursive path models. For each endogenous (dependent) variable in the model, obtain the successive reduced-form equations, beginning with that containing only exogenous (pre-determined) variables, then adding intervening variables in sequence from cause to effect. The total effect of a variable is its coefficient in the first reduced-form equation in which it appears as a regressor, that is, the equation in which the regressors are the causal variable of interest and other contemporaneous or causally prior variables. Of course, the sum of noncausal components of association may be found as the difference between a total effect and the corresponding zero-order measure of association. Indirect components of effects are given by differences between coefficients of a causal variable in two equations in the sequence, where the mediating variable (or variables) is that which appears as regressor in one equation and not in the other. However, it must be understood that indirect effects so computed will include those mediated by the intervening variable(s) in question and later variables. Finally, the direct effect of a variable is given by its coefficient in the last (structural) equation in the

Table 1. Coefficients of reduced-form and structural equations for variable  $X_3$  in symbolic form.

Predetermined Variable	Equation		
	(1)	(2)	(3)
$X_a$	$q_{3a}$	$q_{3a}^*$	$p_{3a}$
$X_b$	$q_{3b}$	$q_{3b}^*$	$p_{3b}$
$X_c$	$q_{3c}$	$q_{3c}^*$	$p_{3c}$
$X_1$		$q_{31}$	$p_{31}$
$X_2$			$p_{32}$

Note: See text for definition of symbols.

sequence. This method will provide an exact accounting of effects whenever the model is fully recursive, that is, when all possible paths are drawn between variables which are causally ordered in the model.

*Tabular Presentation of Coefficients*

Consider the coefficients from the series of equations for  $X_3$  which are given in Table 1. Each column of Table 1 gives the coefficients of a reduced-form or structural equation. The rows refer to regressors which enter the equations. Column 1, for example, gives the coefficients in the regression of  $X_3$  on  $X_a$ ,  $X_b$ , and  $X_c$ , and row one contains the coefficients for  $X_a$  in each of the three equations for  $X_3$ . Taking the coefficients for  $X_a$ , let us briefly summarize the information conveyed by the table. First,  $q_{3a}$  represents the total effect of  $X_a$  on  $X_3$ . Second,  $q_{3a} - q_{3a}^*$  is the effect of  $X_a$  which operates via  $X_1$  (and later variables). Third,  $q_{3a}^*$  is the effect of  $X_a$  not mediated by  $X_1$ . Fourth,  $q_{3a}^* - p_{3a}$  is the effect unmediated by  $X_1$  but mediated by  $X_2$ . Fifth,  $q_{3a} - p_{3a}$  is the effect mediated by  $X_1$ ,  $X_2$  or both. Finally,  $p_{3a}$  is the effect which is unmediated by  $X_1$  and/or  $X_2$ .

If we express the above effects as proportions of the total effect we have the following quantities:

- (1)  $1 - q_{3a}^*/q_{3a}$  —the proportion of the total effect mediated by  $X_1$ .

- (2)  $q_{3a}^*/q_{3a}$  —the proportion of the total effect unmediated by  $X_1$ .

- (3)  $(q_{3a}^* - p_{3a})/q_{3a}$  —the proportion of the total effect unmediated by  $X_1$  which is mediated by  $X_2$ .

- (4)  $1 - p_{3a}/q_{3a}$  —the proportion of the total effect mediated by  $X_1$  and/or  $X_2$ .

- (5)  $p_{3a}/q_{3a}$  —the proportion of the total effect unmediated by  $X_1$  and/or  $X_2$ .

It should be clear from the above that the quantities in (1) and (2) sum to unity; the quantities in (4) and (5) sum to unity; and the quantities in (1), (3), and (5) sum to unity.<sup>3</sup>

*Numerical Illustration*

To illustrate the interpretive scheme we use published results for a recursive model of socioeconomic achievement which (not coincidentally) has the same structure as the model in Figure 1. The model and data are from Duncan, Featherman and Duncan (1972:37-45). The variables are  $X_a$  = father's occupational status,  $X_b$  = father's education,  $X_c$  = number of siblings,  $X_1$  = son's education,  $X_2$  = son's occupational status, and  $X_3$  = son's income. The data pertain to non-black men of nonfarm background in the U.S. experienced civilian labor force at ages 35 to 44 in March, 1962. In Table 2 we present estimated coefficients of regression equations used to interpret the causal model in Figure 1. The first regression equation is for variable

<sup>3</sup>Occasionally one will encounter situations in which direct and indirect effects counteract one another, i.e., suppressor effects, so the total effect is less than the sum of the absolute effects, and some components may be larger than the total effect. If one wishes to express the direct and indirect effects as proportions of the total effect in such a case, the sum of the unsigned proportions will exceed unity. A possible solution is to express the various components as proportions of the sum of their absolute values.



Table 2. Coefficients of variables in standard form in reduced-form and structural equations of a model of socioeconomic achievement: non-black U.S. men with non-farm background in the experienced civilian labor force, aged 35 to 44 in March, 1962.

Predetermined Variable	Equation and dependent variable					
	(1) $X_1$	(2) $X_2$	(3) $X_2$	(4) $X_3$	(5) $X_3$	(6) $X_3$
$X_a$	.2781	.2842	.1266	.1746	.0903	.0492
$X_b$	.1985	.1199	.0074	.1120	.0518	.0494
$X_c$	-.2053	-.1703	-.0540	-.0998	-.0376	-.0200
$X_1$			.5668		.3033	.1193
$X_2$						.3247

Note: See text for definition of symbols. Based on correlations in Duncan, Featherman, and Duncan (1972:38).

$X_1$ , the second and third for  $X_2$ , and the fourth through sixth are for  $X_3$ .

In Table 3 we rearrange the coefficients of Table 2, following the pattern described above, in order to interpret the effects of each variable in the model. We believe this form of presentation communicates the causal importance of variables unambiguously. Not only are the effects of variables communicated in a straightforward manner, but the mechanisms by which these effects come about are conveyed as well. For example,  $X_a$  has an effect of .1746 on  $X_3$ , of which .0843 (48 percent) is transmitted via  $X_1$ , .0411 (24 percent) is transmitted via  $X_2$ , and .0492 (28 percent) is unmediated by variables in the model. Further, in order to understand how a particular intervening variable exercises its effect, we may decompose the indirect effects into their constituent parts. For example, of the effect transmitted by  $X_1$  (.0843),  $.0331 = .0843 \times .1193/.3033$  (19 percent of the total effect) is transmitted directly, and  $.0512 = .0843 \times .1840/.3033$  (29 percent of the total effect) is transmitted via  $X_1$ 's effect on  $X_2$  and its subsequent effect on  $X_3$ .

Thus, of the effect of father's occupational status on son's income, almost half is due to the educational advantage of sons with high status fathers. Another quarter is explained by the higher occupational status of sons with high status fathers. The remaining quarter of the effect of father's status on son's income is direct; that is, it is not explained either by the greater schooling or higher status jobs of men with high status fathers. Further, about 60 percent of the effect of father's status by way of son's schooling is due to the higher status jobs of more educated sons, and the remainder is due to the direct effect of schooling on income. We shall not offer further substantive interpretation of these results, which is given in the source publication.

### *Summary and Discussion*

Following Duncan (1971) and Finney (1972) (see also Lewis-Beck, 1974) we have indicated that correlations among variables in a recursive model may contain one or more of the following components: correlation involving unanalyzed association among pre-

determined variables, correlation due to the joint dependence of the variables on common or correlated variables (spurious correlation), and effects. We have drawn attention to the necessary distinction between total effects and total associations, and reemphasized the definition of indirect effects as effects which are transmitted via intervening variables.

Next, we looked at the interpretation of effects in recursive models and suggested a method for obtaining total effects of variables which is often easier than computing them from a path diagram. The total effect of one variable on another in a recursive model can be obtained by estimating the regression equation which contains only the variable whose effect is desired plus the variables prior to it and contemporary with it in the model. Although we have dealt exclusively with standardized variables, the principle is invariant with regard to metric. We have shown how this principle can be applied by successively estimating reduced-form equations in a recursive model to generate total, indirect and direct effects of variables in the model. The successive computation of reduced-form equations for a particular dependent variable begins with a model which contains only exogenous variables in the system, then successively adds the variables (or sets of variables) which intervene, proceeding in sequence from cause to effect until the intervening variables are exhausted. This generates all the information required to decompose total effects into their various mediated and unmediated parts. It capitalizes on the fact that when an intervening variable (or set of variables) is added in the sequence, the indirect effect(s) which operates via that variable is removed from the coefficients of other variables. We have also shown how to present this decomposition in a table which summarizes all the relevant information and succinctly presents the detailed interpretation of effects. Similar methods may be used to interpret spurious relationships and in the interpretation of nonrecursive models, but we have not developed those ideas here.

Our formal exposition was developed in terms of population parameters, but the same arguments may be applied to corresponding sample statistics (as in our illustration). Of course, sample estimates are subject to

Table 3. Interpretations of effects in a model of socioeconomic achievement: non-black U.S. men with non-farm background in the experienced civilian labor force, aged 35 to 44 in March, 1962.

Dependent variable	Predetermined variable	Total effect	$\frac{\text{Indirect effects via } X_1}{X_2}$	Direct effect
X <sub>1</sub>	X <sub>a</sub>	.2781	--	.2781
	X <sub>b</sub>	.1985	--	.1985
	X <sub>c</sub>	-.2053	--	-.2053
X <sub>2</sub>	X <sub>a</sub>	.2842	.1576	.1266
	X <sub>b</sub>	.1199	.1125	.0074
	X <sub>c</sub>	-.1703	-.1163	-.0540
X <sub>3</sub>	X <sub>1</sub>	.5668	--	.5668
	X <sub>a</sub>	.1746	.0843	.0492
	X <sub>b</sub>	.1120	.0602	.0494
	X <sub>c</sub>	-.0998	-.0622	-.0200
	X <sub>1</sub>	.3033	--	.1840
	X <sub>2</sub>	.3247	--	.3247

Note: See text for definitions of symbols. Data from Table 2.

sampling variability, but we have not attempted to deal here with issues of statistical inference. Standard inferential techniques will apply to decisions about the relative magnitudes of components of effects within one population or about the equivalence of proportionate decompositions across populations.

Obviously, we believe it is important to interpret patterns of direct and indirect causation in path models and other structural equation models. Such interpretations help us answer questions of the form, "How does variable X affect variable Y?," or "How much does mechanism Z contribute to the effect of X on Y?," or "Does mechanism Z contribute as much to explaining the effect of X on Y in population A as in population B?" At the same time, we should be disappointed if our efforts to elucidate such causal interpretations were to lead researchers to generate vast quantities of uninteresting or meaningless components. Sometimes a detailed interpretation will speak to an important research question, and at other times it will not. We offer no substitute for the thoughtful interpretation of social data.

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